PMM U.S.S.R.,Vol.46,pp. 388-394 0021-8928/83/3 0388$7.50/0

UDC 539.3+517.948

METHOD OF NUMERICAL SOLUTION OF THE PROBLEM OF IMPRESSING A MOVING STAMP INTO AN ELASTIC HALF-PLANE, TAKING HEAT GENERATION INTO ACCOUNT*

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The contact problem of impressing a uniformly moving stamp of arbitrary configuration into an elastic half-plane is examined in the presence of heat generation because of tangential stress. The usual thermal and elastic contact conditions reduce the solution of the problem posed to the solution of a system of singular integral equations of the first and second kind. Consequently, the unknown functions in the system possess different kinds of singularities at the ends. A single method of numerically solving such equations that is convenient for application on electronic computers is proposed and given a foundation. This method is to make the transition from a singular integral equation to a system of linear algebraic equations in values of the required functions in roots of appropriate Jacobi polynomials. The heat flux distribution and their influence on the contact pressure distribution are investigated on the basis of the method proposed.

The contact problem of impressing a stamp moving at a constant velocity along a boundary, into an elastic half-plane was first formulated and solved in the monograph /1/. Analogous problems were examined in /2,3/ for systems of stamps. However, tangential stresses under the stamp that cause heat generation were not taken into account in these papers, as is apparently essential in real cases.

To solve problems taking the mentioned effects into account, as well as other problems reducing to similar equations, a method of numerical solution of singular integral equations of the first and second kind that is convenient for application of electronic computers is necessary. For equations of the first kind, such a method was developed in aerodynamics ("the method of discrete vortices") by using heuristic considerations and methodological computations on an electronic computer /4/. Its mathematical foundation is given in /5 — 7/. An analogous method for equations of the first kind is proposed, without mathematical foundation, in the theory of cracks /8/ by using Chebyshev polynomials and giving a higher rate of convergence than the "method of discrete vortices". The single method proposed below for the numerical solution of singular integral equations of the first and second kinds is a generalization of the above-mentioned methods.

1. Let a stamp compressed on a half-plane by a force P move along the boundary of the elastic half-plane at a constant velocity V_0 less than the velocity of shear wave propagation in the elastic half-plane. Because of dry friction, a quantity of heat proportional to the velocity of stamp motion, to the friction coefficient, and to the normal contact pressure /9, 10/, is generated in the contact zone. We assume that the stamp dimensions considerably exceed the length of the contact zone, whereupon it can be replaced by the half-plane for the determination of the stamp temperature.

We select a fixed $O_1X_1Y_1$ coordinate system and a moving OXY coordinate system rigidly clamped to the stamp and determined by the formulas $x = x_1 - V_0t,\ y = y_1$. The heat generation in the contact zone results in the appearance of the heat fluxes $Q_1'(x)$ and $Q_2'(x)$ directed, respectively, into the half-plane and the stamp, and connected with the contact pressure $p'(x)$ by the relationship

$$Q_1'(x) + Q_2'(x) = c\beta V_0 p'(x) \tag{1.1}$$

where c is the thermal equivalent of the mechanical work, and β is the friction coefficient. According to the Fourier law of heat conduction, we have

$$Q_k'(x) = (-1)^{k+1}\lambda_k \frac{\partial T_k'}{\partial y}, \quad k = 1, 2$$

where λ_1 and λ_2 are heat conduction coefficients, and T_1' and T_2' are, respectively, the half-plane and stamp temperatures. The parts of the stamp and half-plane surfaces, not in contact,

*Prikl.Matem.Mekhan.,46,No.3,pp.494-501,1982 are assumed heat-insulated.

We use the results of /11/, where expressions were obtained for the displacement and temperature of boundary points of an elastic strip, particularly a half-plane, when lumped forces and thermal sources move at the constant velocity V_0 along its edges. On the basis of the superposition principle, expressions can hence be obtained directly for the components mentioned for distributed forces and heat sources. In dimensionless variables we have

$$v(\xi, 0) = \frac{k_1(1-k_2^2)}{\pi\Delta}\int \ln\frac{1}{|\xi - s|}\cdot p(s)\,ds + C_1^* + \tag{1.2}$$

$$\frac{2k_1k_2-(1+k_2^2)}{2\Delta}\int \operatorname{sign}(\xi - s)\,q(s)\,ds - \frac{(1-k_1^2)(1+k_2^2)}{\Delta}\int K(\xi - s)\,Q_1(s)\,ds$$

$$T_1(\xi, 0) = \frac{1}{\pi}\int \ln|\xi - s|\,Q_1(s)\,ds + C_2^* - \int R(\xi - s)\,Q_1(s)\,ds \tag{1.3}$$

$$v = \frac{\mu}{P}v'(x, y), \quad T(\xi) = \frac{\gamma a}{P}T'(x), \quad \xi = \frac{x}{a}$$

$$p(\xi) = \frac{a}{P}p'(x), \quad q(\xi) = \frac{a}{P}q'(x), \quad Q_i(\xi) = \frac{P\cdot\lambda_i}{\gamma a^2}Q_i'(x), \quad i = 1, 2$$

$$K(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{k_1|s|}{\sqrt{s^2+is\varkappa}} - 1\right]\frac{e^{-isu}}{s^2(1-k_1^2)+is\varkappa}\,ds$$

$$R(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{1}{\sqrt{s^2+is\varkappa}} - \frac{1}{|s|}\right]e^{-isu}\,ds$$

$$\varkappa = \rho c_\varepsilon V_0 a/\lambda_1, \quad \gamma = (3\lambda + 2\mu)\alpha_t$$

$$\Delta = 4k_1k_2 - (1+k_2^2)^2, \quad k_i^2 = 1 - \frac{V_0^2}{c_i^2}, \quad i = 1, 2$$

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}$$

where λ and μ are Lamé coefficients, ρ is the density, α_t is the coefficient of thermal expansion, c_ε is the specific heat of the half-plane material, a is the half-length of the contact zone, C_1^* and C_2^* are certain, generally infinite, constants. Here and henceforth, if limits are not specified, the integration will be between -1 and 1.

Because of the assumption made, the temperature of the boundary points of the stamp is determined by the formula (C_3^* is a certain, also infinite, constant)

$$T_2(\xi, 0) = \frac{1}{\pi}\int \ln|\xi - s|\,Q_2(s)\,ds + C_3^* \tag{1.4}$$

If a harmonic regime, in time, with frequency w is considered instead of the stationary regime of the temperature distribution in the stamp and the half-plane, and a wave going to infinity is liberated, then by equating the temperature of the stamp (1.4) and the half-plane (1.3) in the contact zone, we obtain a thermal contact equation in the limit $w \to 0$:

$$\int \ln|\xi - s|\,Q_1(s)\,ds - \pi\int R(\xi - s)\,Q_1(s)\,ds = \int \ln|\xi - s|\,Q_2(s)\,ds \tag{1.5}$$

and a condition for which the infinite constants C_2^* and C_3^* mutually cancel out

$$\int Q_1(s)\,ds = \int Q_2(s)\,ds \tag{1.6}$$

We differentiate (1.5) with respect to ξ and eliminate $Q_2(s)$ by using the condition (1.1). We consequently obtain the equation

$$(1+\lambda)\int \frac{Q_1(s)}{s-\xi}\,ds - \zeta\int \frac{p(s)}{s-\xi}\,ds + \pi\int \frac{\partial R(\xi - s)}{\partial \xi}Q_1(s)\,ds = 0 \tag{1.7}$$

$$\lambda = \lambda_1/\lambda_2, \quad \zeta = \gamma c\beta V_0 a/\lambda_2$$

Condition (1.6) becomes

$$\int Q_1(s)\,ds = \frac{\zeta}{1+\lambda} \tag{1.8}$$

where the equilibrium condition of the stamp is also used

$$\int p(s)\,ds = 1 \tag{1.9}$$

Let us turn to an examination of the elastic contact between the stamp and the half-plane. We have the usual condition /12/

$$v(\xi, 0) = f(\xi) - d$$

where $f(\xi)$ is a function describing the base of the stamp, and d is the dimensionless settlement of the stamp. We substitute (1.2) into the last condition and we take into account that $q(\xi) = \beta p(\xi)$ in the contact zone. Differentiating the equation obtained with respect to ξ, we will have

$$\int \frac{p(s)\,ds}{s-\xi} + \pi\beta\vartheta_0 p(\xi) + \vartheta_2 \int \frac{\partial K(\xi - s)}{\partial \xi} Q_1(s)\,ds = -\frac{1}{\vartheta_1} f'(\xi) \qquad (1.10)$$

$$\vartheta_0 = \frac{1 + k_2^2 - 2k_1k_2}{k_1(1-k_2^2)}, \qquad \vartheta_1 = \frac{k_1(1-k_2^2)}{\pi\Delta}, \qquad \vartheta_2 = \frac{\pi c_2^2(1+k_2^2)}{k_1 c_1^2}$$

Therefore, (1.7) and (1.10), together with (1.8) and (1.9), form a complete system of singular integral equations in the unknown heat flux $Q_1(\xi)$ and contact pressure $p(\xi)$.

It is convenient to introduce the new unknown functions

$$\chi(\xi) = (1+\lambda) Q_1(\xi) - \zeta p(\xi), \quad \psi(\xi) = p(\xi)$$

We afterwards obtain the system

$$\int \frac{\chi(s)}{s-\xi}\,ds + \frac{\pi}{1+\lambda} \int \frac{\partial R(\xi - s)}{\partial \xi} [\chi(s) + \zeta\psi(s)]\,ds = 0 \qquad (1.11)$$

$$\int \frac{\psi(s)}{s-\xi}\,ds + \pi\beta\vartheta_0\psi(\xi) + \frac{\pi\vartheta_2}{1+\lambda} \int \frac{\partial K(\xi - s)}{\partial \xi} [\chi(s) + \zeta\psi(s)]\,ds = -\frac{f'(\xi)}{\vartheta_1}$$

under the conditions

$$\int \chi(s)\,ds = 0, \qquad \int \psi(s)\,ds = 1$$

The system (1.11) contains singular integral equations of the first and second kinds. A method for numerical solution of such equations, which is convenient for application of electronic computers, is proposed and given a foundation below.

2. We consider the singular integral equation (a and b are real numbers)

$$a\gamma(x_0) + \frac{b}{\pi} \int \frac{\gamma(x)}{x - x_0}\,dx = f(x_0) \qquad (2.1)$$

$$x_0 \in (-1, 1), \ a^2 + b^2 = 1, \ b \neq 0, \ f(x) \in H(\alpha)$$

It is known /13/ that the index $\varkappa$ of (2.1) takes on the values $1, 0, -1$ while the corresponding solutions have the form

$$\gamma(x) = w(x)\,\varphi(x), \qquad (2.2)$$

$$(w(x) = (1-x)^\alpha (1+x)^\beta, \quad 0 < |\alpha|, \ |\beta| < 1,$$

$$\varkappa = -(\alpha + \beta))$$

The number α is determined by the equation

$$a + b \operatorname{ctg} \pi\alpha = 0 \qquad (2.3)$$

Let $I(x_0)$ denote the left side of (2.1), and let the formula

$$I_n(x_0) = a\gamma_n(x_0) + \frac{b}{\pi} \int \frac{\gamma_n(x)}{x - x_0}\,dx, \quad \gamma_n(x) = \omega(x)\,\varphi_n(x), \quad \varphi_n(x) = \sum_{i=1}^{n} \frac{\varphi_n(x_i)\,P_n^{(\alpha,\beta)}(x)}{(x - x_i)\,P_n'^{(\alpha,\beta)}(x_i)}$$

be the quadrature-interpolation formula of index $\varkappa$ of order n for this function, where x_i ($i = 1, \ldots, n$) are roots of the Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$ of degree n corresponding to the function $\omega(x)$, $\varphi_n(x_i) = \varphi(x_i)$.

The polynomial $P_n^{(\alpha,\beta)}(x)$ satisfies the relationship /14/

$$a\omega(x_0)\,P_n^{(\alpha,\beta)}(x_0) + \frac{b}{\pi} \int \frac{\omega(x)\,P_n^{(\alpha,\beta)}(x)}{x - x_0}\,dx = -2^{-\varkappa} \frac{b}{\pi} \Gamma(\alpha)\Gamma(1-\alpha)\,P_{n-\varkappa}^{(-\alpha,-\beta)}(x_0). \qquad (2.4)$$

Hence, the following equality is valid

$$I_n(x_0) = \sum_{i=1}^{n} \frac{\varphi_n(x_i)}{(x_0 - x_i)\,P_n'^{(\alpha,\beta)}(x_i)} \left[-2^{-\varkappa} \frac{\Gamma(\alpha)\Gamma(1-\alpha)}{\pi} b P_{n-\varkappa}^{(-\alpha,-\beta)}(x_0) - \frac{b}{\pi} \int \frac{\omega(x)\,P_n^{(\alpha,\beta)}(x)}{x - x_i}\,dx\right]$$

We designate the function $\gamma_n(x)$ the approximate solution of (2.1) and find it from the equality of the functions $I_n(x_0)$ and $f(x_0)$. The function $\gamma_n(x)$ will be defined if we find the numbers $\varphi_n(x_i)$ ($i = 1, \ldots, n$) with respect to which it is desirable to have a convenient system of n linear algebriac equations. Equating the functions $I_n(x_0)$ and $f(x_0)$ at the points x_{0j} ($j = 1, \ldots, n - \varkappa$), where x_{0j} are the roots of the polynomial $P_{n-\varkappa}^{(-\alpha,-\beta)}(x)$, we obtain the following system of linear equations

$$-\frac{b}{\pi}\sum_{i=1}^{n}\frac{\varphi_n(x_i)\,a_i}{x_{0j}-x_i}=f(x_{0j}),\qquad j=1,\ldots,n-\varkappa \tag{2.5}$$

$$a_i=\frac{1}{P_n'^{(\alpha,\beta)}(x_i)}\int\frac{\omega(x)\,P_n^{(\alpha,\beta)}(x)}{x-x_i}\,dx$$

Setting $x_0=x_i$ in (2.4), we note that the coefficient a_i in (2.5) can be written in the form

$$a_i=-2^{-\varkappa}\Gamma(\alpha)\Gamma(1-\alpha)\frac{P_{n-\varkappa}^{(-\alpha,-\beta)}(x_i)}{P_n'^{(\alpha,\beta)}(x_i)},\qquad i=1,\ldots,n \tag{2.6}$$

Let us consider different values of the index $\varkappa$.

Let $\varkappa=0$. In this case, the unique solution of (2.1) is extracted by giving the number α, $0<|\alpha|<1$ satisfying (2.3). If it is necessary to obtain the solution bounded at the point 1 and unbounded at the point -1, then a positive number α should be taken. The polynomials $P_n^{(\alpha,\beta)}(x)$ and $P_{n-\varkappa}^{(-\alpha,-\beta)}(x)$ have an identical number of roots, hence, the system (2.5) contains n unknowns and n equations.

Let $\varkappa=1$. In this case $-1<\alpha,\beta<0$, and the equation (2.1) has no unique solution. An unique solution can be extracted by the additional condition

$$\int\gamma(x)\,dx=C \tag{2.7}$$

In this case the polynomial $P_{n-\varkappa}^{(-\alpha,-\beta)}(x)$ has the degree $(n-1)$, hence the system (2.5) contains n unknowns and $(n-1)$ equations, i.e., it is indeterminate. Let us determine it by using discretization of (2.7), i.e., we consider the system

$$-\frac{b}{\pi}\sum_{i=1}^{n}\frac{\varphi_n(x_i)\,a_i}{x_{0j}-x_i}=f(x_{0j}),\qquad j=1,\ldots,n-1 \tag{2.8}$$

$$\sum_{i=1}^{n}\varphi_n(x_i)\,a_i=C,\ j=n$$

Now, let $\varkappa=-1$. In this case the polynomial $P_{n-\varkappa}^{(-\alpha,-\beta)}(x)$ has $(n+1)$ roots, consequently, the system (2.5) is over-determined and, as a rule, incompatible. Hence, exactly the same as in /6,7/ we consider the system

$$\gamma_{0n}-\frac{b}{\pi}\sum_{i=1}^{n}\frac{\varphi_n(x_i)\,a_i}{x_{0j}-x_i}=f(x_{0j}),\qquad j=1,\ldots,n+1 \tag{2.9}$$

where γ_{0n} is the regularizing variable /15/. In case $\varkappa=-1$, it is also known that the solution exists only upon satisfying the condition (α is determined by (2.3)):

$$\int\frac{f(x)}{(1-x)^{\alpha}(1+x)^{\beta}}\,dx=0;\quad 0<\alpha,\quad \beta<1,\quad \alpha+\beta=1 \tag{2.10}$$

As in /7/, it is shown that the system (2.5) for $\varkappa=0$, and the systems (2.8) and (2.9) are not degenerate, and also that $\gamma_{0n}\to 0$ as $n\to\infty$ if and only if condition (2.10) is satisfied.

We have for values of $\varphi_n(x_i)$ /7/

$$\varphi_n(x_i)=\frac{1}{a_i}I_{\varkappa,i}^{(n)}\left[\sum_{j=1}^{n-\varkappa}I_{\varkappa,0j}^{(n)}\frac{-b\pi^{-1}f(x_{0j})}{x_i-x_{0j}}+\nu_\varkappa C\right],\quad i=1,\ldots,n \tag{2.11}$$

$$I_{\varkappa,i}^{(n)}=\prod_{m=1}^{n-\varkappa}(x_i-x_{0m})\left[\prod_{\substack{m=1\\ m\neq i}}^{n}(x_i-x_m)\right]^{-1}$$

$$I_{\varkappa,0j}^{(n)}=\prod_{m=1}^{n}(x_{0j}-x_i)\left[\prod_{\substack{m=1\\ m\neq j}}^{n-\varkappa}(x_{0j}-x_{0m})\right]^{-1}$$

$$\nu_1=1,\ \nu_0=\nu_{-1}=0$$

Because of the representation of polynomials by the product of linear factors, and of (2.6), we obtain

$$I_{\varkappa, i}^{(n)} = -\frac{2^{\varkappa} a_i}{\Gamma(\alpha)\Gamma(1-\alpha)} \frac{P_n^{(\alpha,\beta)}}{P_{n-\varkappa}^{(-\alpha,-\beta)}}, \qquad i = 1, \ldots, n$$

where $P_n^{(\alpha,\beta)}$ and $P_{n-\varkappa}^{(-\alpha,-\beta)}$ are coefficients of the highest powers of the unknown in the appropriate Jacobi polynomials.

It is known /13/ that the function $\varphi(x)$ in (2.2) for the solution of the index $\varkappa$, is determined by the formula

$$\varphi(x) = a\frac{f(x)}{\omega(x)} - \frac{b}{\pi}\int \frac{f(x_0)\,dx_0}{\omega(x_0)(x_0 - x)} + T_{\varkappa} C \tag{2.12}$$

$$T_1 = -[\Gamma(\alpha)\Gamma(1-\alpha)]^{-1}, \qquad T_0 = T_{-1} = 0$$

We consider equality (2.12) as an equation for the function $\eta(x) = [\omega(x)]^{-1} f(x)$. If $\gamma(x)$ is a solution of index $\varkappa$ for (2.1), then the function $\eta(x)$ will be a solution of index $-\varkappa$ for (2.12) /13/. If we replace x by x_0, $\omega(x)$ by $1/\omega(x)$, and $P_n^{(\alpha,\beta)}(x)$ by $P_{n-\varkappa}^{(-\alpha,-\beta)}(x)$ in (2.4), then the number b must be replaced by $-b$. We consequently obtain

$$a\frac{P_{n-\varkappa}^{(-\alpha,-\beta)}(x)}{\omega(x)} - \frac{b}{\pi}\int \frac{P_{n-\varkappa}^{(-\alpha,-\beta)}(x_0)\,dx_0}{x_0 - x} = 2^{\varkappa}\frac{\Gamma(-\alpha)\Gamma(1+\alpha)}{\pi} bP_n^{(\alpha,\beta)}(x)$$

Now let $\Phi(x)$ denote the function obtained from $I(x_0)$ by replacing $\gamma(x)$ by $\eta(x)$, b by $-b$, x_0 by x. If $\Phi_{n-\varkappa}(x)$ denotes the function obtained from $\Phi(x)$ by replacing $\eta(x)$ by $\eta_{n-\varkappa}(x)$, where $\eta_{n-\varkappa}(x)$ is determined by the roots x_{0j} $(j = 1, \ldots, n-\varkappa)$ of the polynomial $P_{n-\varkappa}^{(-\alpha,-\beta)}(x)$ is analogous to the function $\gamma_n(x)$, then we obtain

$$\Phi_{n-\varkappa}(x_i) = \frac{b}{\pi}\sum_{j=1}^{n-\varkappa} \frac{f(x_{0j}) b_j}{x_i - x_{0j}}, \qquad i = 1, \ldots, n, \qquad b_j = -2^{\varkappa}\Gamma(-\alpha)\Gamma(1+\alpha)\frac{P_n^{(\alpha,\beta)}(x_{0j})}{P_n'^{(\alpha,\beta)}(x_{0j})}$$

at the points x_i $(i = 1, \ldots, n)$ which are roots of the polynomial $P_n^{(\alpha,\beta)}(x)$.

If again the representation of polynomials by a product of linear factors is used, then we obtain

$$I_{\varkappa, 0j}^{(n)} = -\frac{2^{-\varkappa} b_j}{\Gamma(-\alpha)\Gamma(1+\alpha)} \frac{P_{n-\varkappa}^{(-\alpha,-\beta)}}{P_n^{(\alpha,\beta)}}, \qquad j = 1, \ldots, n-\varkappa$$

Finally, we note that the equalities

$$\frac{P_n^{(\alpha,\beta)}}{P_{n-\varkappa}^{(-\alpha,-\beta)}} = 2^{-\varkappa}; \qquad \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)\Gamma(-\alpha)\Gamma(1+\alpha)} = -\frac{b^2}{\pi^2}$$

are valid /16/. We thus obtain

$$\varphi_n(x_i) = \frac{b}{\pi}\sum_{j=1}^{n-\varkappa} \frac{f(x_{0j}) b_j}{x_i - x_{0j}} - \frac{\nu_{\varkappa}}{\Gamma(\alpha)\Gamma(1-\alpha)} C, \quad i = 1, \ldots, n \tag{2.13}$$

It is seen that $-[\Gamma(\alpha)\Gamma(1-\alpha)]^{-1}\nu_{\varkappa} = T_{\varkappa}$. Therefore, (2.11) and (2.13) yield approximate values of the function $\varphi(x)$ defining the solution $\gamma(x)$. If $f(x)$ is a polynomial of degree $(n-\varkappa)$, then we obtain an exact value of the solution $\gamma(x_i) = \omega(x_i)\varphi_n(x_i)$. If $f'(x) \in H(\alpha)$, then it follows from the results of /17/

$$|\varphi(x_i) - \varphi_n(x_i)| \leqslant O(E'_{n-\varkappa-1}), \quad i = 1, \ldots, n$$

where $E'_{n-\varkappa-1}$ is the best approximation of the function $f'(x)$ by polynomials of degree $(n-\varkappa-1)$.

Now we consider the equation

$$a\gamma(x_0) + \frac{b}{\pi}\int \frac{\gamma(x)}{x - x_0}\,dx + \int K(x, x_0)\gamma(x)\,dx = f(x_0) \tag{2.14}$$

where $K(x, x_0) \in H(\alpha)$ in $[-1,1] \times [-1,1]$. This equation also has a solution of index $\varkappa = 1, 0, -1$, and is equivalent to the corresponding Fredholm integral equation of the second kind in each of the classes of solutions. Consequently, all the results obtained above for (2.1) are valid for this equation also. Systems of linear algebraic equations for (2.14) are obtained from the corresponding systems for (2.1) by appending the term

$$\sum_{i=1}^{n} K(x_i, x_{0j})\varphi_n(x_i) a_i, \qquad j = 1, \ldots, n-\varkappa$$

The results formulated above are also valid for equations of the first kind, i.e., when we have $a = 0$ in (2.1) and (2.14). In this case α and β take on the values $\pm 1/2$ and the corresponding Jacobi polynomials are expressed in terms of Chebyshev polynomials of the first and second kinds.

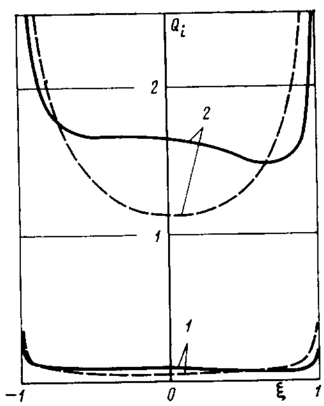

Fig.1

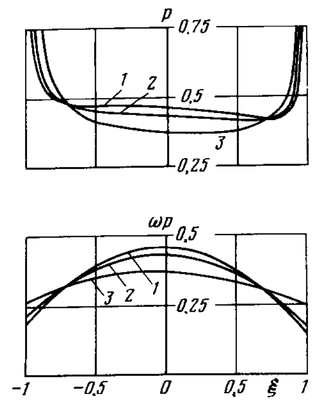

Fig.2

3. The system (1.11) was analyzed numerically for different stamp motion velocities on the basis of the method elucidated in Sect.2, for the following values of the parameters:

$$\lambda = 35,\ \xi = 642\ V_0/c_2,\ c_2^2/c_1^2 = 0.275,$$
$$\beta = 0.27,\ \varkappa = 1.807 \cdot 10^{-5}\ V_0/c_2$$

and the profile of the stamp base $f(x) = 0.1\ x^2$.

The solid lines in Fig.1 show the heat flux distribution $Q_1 = P\lambda_1 (\gamma a^2)^{-1} Q_1'$ directed into the half-plane for the velocities $V_0/c_2 = 0.01$ (curve *1*) and $V_0/c_2 = 0.2$ (curve *2*), and the dashes are the heat flux distribution laws $Q_2 = P\lambda_2 (a^2\gamma)^{-1} Q_2'$ directed into the stamp at the same velocities.

The values of the contact pressure at the points that are roots of the Jacobi polynomial $P_{10}^{(-\alpha,\alpha-1)}(x)$ ($\alpha = \pi^{-1} \operatorname{arcctg} \beta\vartheta_0$) are compared with values of the function

$$\frac{a}{P} p(x) = \frac{1}{(1-x)^{\alpha}(1+x)^{1-\alpha}} \cdot \frac{\sin \pi\alpha}{5\pi\vartheta_1} [5\vartheta_1 + 2\alpha(1-\alpha) - (1-2\alpha)x - x^2]$$

at these same points, which is an exact analytical expression for the contact pressure for an analogous problem without heat generation taken into account, as obtained by using the method of orthogonal polynomials. Comparison shows that the heat being liberated, although significant in magnitude (Fig.1), has practically no influence on the contact pressure distribution (the difference does not exceed 0.03% for $V_0/c_2 = 0.2$).

Contact pressure distributions and their regular parts are presented in Fig.2 for different velocities of stamp motion (the values $V_0/c_2 = 0.44 \cdot 10^{-5}$; 0.5; 0.8 correspond to curves *1*, *2*, *3*). It is seen that as the velocity increases, the pressure drops in the middle part of the contact zone, and the stress concentration coefficients grow at the ends of the contact zone.

The authors are grateful to S.M. Mkhitarian for formulating the problem.

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Translated by M.D.F.